

$N = 1$ super-Chern-Simons theory in Batalin-Vilkovisky formulation

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We analyse the Abelian $N = 1$ super-Chern-Simons model coupled to parity-preserving matter in linear and non-linear gauges with exact BRST invariance. Then we analyse the theory in field/antifield formulation to discuss the model at quantum level. Furthermore, we implement the field/antifield dependent transformation parameter to generalize the BRST symmetry of the theory. The novelty of field/antifield dependent BRST transformation is that under change of variable the Jacobian of the functional integral extends the quantum action from linear gauge to non-linear gauge. The results are established in full generality.

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I. INTRODUCTION

The supersymmetric as well as the ordinary version of Abelian gauge field theories in three dimensional (3D) space-times are subject of enormous interest in a recent past [1–3]. At low energies, supersymmetric Chern-Simons theories are important because they describe the world-volume of $M2$ -membranes in M -theory [4–8]. In fact, the world-volume of $M2$ -membranes in M -theory, at low energies, is thought to be described by the $N = 8$ superconformal Chern-Simons-matter theory [9]. However, Chern-Simons theory with $N = 1$ supersymmetry has also been studied in relation to axion gauge symmetry [10]. Besides their relevance in connection with the possibility of getting non-perturbative results, the recent results on the Landau gauge finiteness of Chern-Simons theories are remarkable that make 3D gauge theories so attractive [11–13]. The gauge theories in 3D space-times provide a basis to tackle exciting topics of condensed matter physics such as high temperature superconductivity and fractional quantum hall effect [14]. It is well known that the Landau gauge has very special features as compared to generic linear or non-linear gauge-fixings [15, 16]. These gauge conditions can be incorporated to the theory at quantum level by adding the suitable gauge-fixing and ghost terms to the classical action which remains invariant under the fermionic rigid BRST invariance [17]. However, the BRST symmetry plays an important role in proof of unitarity and renormalizability of the gauge theories [18, 19].

On the other hand, the Batalin-Vilkovisky (BV) formulation is a more fundamental approach to quantize the more general gauge theories with open gauge algebra such as the supergravity and topological field theories [18–24]. Subsequently, the generalization of BRST transformations by making the infinitesimal parameter finite and field dependent is originally advocated in [25] and has found several applications in gauge field theories [26–35].

Our aim is to connect the different solutions (extended quantum actions) of the quantum master equation for the $N = 1$ super-Chern-Simons theory. We establish the results which hold up to all order of corrections. For this purpose, we first investigate the theory in linear and non-linear gauges with their BRST invariances. Furthermore, the BRST symmetry transformation is made field/antifield dependent. The Jacobian of the path integral measure is evaluated explicitly under change of variables. We show that the solutions of the quantum master equation in BV formulation can be mapped under field/antifield dependent BRST transformation.

The present paper is organized as follows. In the second section, we discuss the $N = 1$ super-Chern-Simons model in linear and non-linear gauges possessing BRST symmetry. In the third section, we analyse

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the theory in the field/antifield formulation which describes the theory at quantum level. Solutions of quantum master equations are connected through field/antifield dependent BRST transformation in the fourth section. The last section summarizes our results.

II. THE MODEL AND THE BRST SYMMETRIES

In this section, we analyse the $N = 1$ Abelian super-Chern-Simons theory with their supersymmetric BRST invariance. For this purpose let us start with the gauge-invariant action for the $N = 1$ super-Chern-Simons theory coupled with matter supermultiplets in a parity-preserving way, in superspace, given by [36–38]

$$\begin{aligned} \Sigma_{inv} = & \frac{1}{2} \int dv \left[k(\Gamma^\alpha W_\alpha) - (\nabla^\alpha \bar{\Phi}_+) (\nabla_\alpha \Phi_+) - (\nabla^\alpha \bar{\Phi}_-) (\nabla_\alpha \Phi_-) + m(\bar{\Phi}_+ \Phi_+ - \bar{\Phi}_- \Phi_-) \right. \\ & \left. - (\bar{\Phi}_+ \Phi_+ - \bar{\Phi}_- \Phi_-)^2 \right], \end{aligned} \quad (1)$$

where the spinorial Majorana superfield Γ_α is the gauge superconnection and matter is represented by the complex scalar superfields Φ_\pm with opposite $U(1)$ -charges. Here the real parameter k has dimension of mass and superspace measure dv has following expression: $dv = d^3x d^2\theta$. The covariant spinorial derivatives for Φ_\pm and their conjugate superfields $\bar{\Phi}_\pm$ are defined as

$$\nabla_\alpha \Phi_\pm = (D_\alpha \mp i\Gamma_\alpha) \Phi_\pm \quad \text{and} \quad \nabla_\alpha \bar{\Phi}_\pm = (D_\alpha \pm i\Gamma_\alpha) \bar{\Phi}_\pm, \quad (2)$$

where spinorial derivative has the following expression: $D_\alpha = \partial_\alpha + i\theta^\beta \partial_{\alpha\beta}$.

Furthermore, we define the superfield-strength for gauge superconnection as

$$W_\alpha = \frac{1}{2} D^\beta D_\alpha \Gamma_\beta. \quad (3)$$

The components of superfields Γ , Φ_\pm and $\bar{\Phi}_\pm$ are defined in terms of spinor derivatives D_α as follows:

$$\begin{aligned} \chi_\alpha &= \Gamma_\alpha|, \quad B = \frac{1}{2} D^\alpha \Gamma_\alpha|, \\ V_{\alpha\beta} &= -\frac{i}{2} D_{(\alpha} \Gamma_{\beta)}|, \quad \lambda_\alpha = \frac{1}{2} D^\beta D_\alpha \Gamma_\beta|, \\ A_\pm(x) &= \Phi_\pm(x, \theta)|, \quad \bar{A}_\pm(x) = \bar{\Phi}_\pm(x, \theta)|, \\ \psi_\pm^\alpha(x) &= D^\alpha \Phi_\pm(x, \theta)|, \quad \bar{\psi}_\pm^\alpha(x) = D^\alpha \bar{\Phi}_\pm(x, \theta)|, \\ F_\pm(x) &= D^2 \Phi_\pm(x, \theta)|, \quad \bar{F}_\pm(x) = D^2 \bar{\Phi}_\pm(x, \theta)|, \end{aligned} \quad (4)$$

where “|” denotes the quantity evaluated at $\theta = 0$. The gauge invariance of the $N = 1$ Abelian super-Chern-Simons action given in (1) [36–38] reflects the redundancy in gauge degree of freedoms. Therefore, one needs to break the local gauge invariance to quantize the theory correctly. There may be many choices for the gauge condition as the physical theory does not depend on the choices of the gauge condition [18]. For the present analysis, we choose the following well established linear (Landau) gauge condition:

$$\Omega_1 := D^\alpha \Gamma_\alpha = 0. \quad (5)$$

This gauge condition can be employed to the theory at quantum level by adding the following gauge-fixing and ghost terms in the action:

$$\Sigma_{gf+gh}^L = \int dv \left[B D^\alpha \Gamma_\alpha + \hat{C} D^2 C \right]. \quad (6)$$

Now, the total effective action in Landau gauge reads

$$\Sigma_{eff}^L = \Sigma_{inv} + \Sigma_{gf+gh}^L, \quad (7)$$

which remains invariant under the following nilpotent BRST transformations:

$$\begin{aligned}\delta_b \Phi_\pm &= \pm i C \Phi_\pm \eta, & \delta_b \bar{\Phi}_\pm &= \mp i C \bar{\Phi}_\pm \eta, \\ \delta_b \Gamma_\alpha &= D_\alpha C \eta, & \delta_b \widehat{C} &= B \eta, \\ \delta_b C &= 0, & \delta_b B &= 0,\end{aligned}\tag{8}$$

where the transformation parameter η is Grassmannian in nature. Furthermore, we restrict the gauge superfield to satisfy another gauge condition which is non-linear (quadratic) in nature as follows

$$\Omega_2 := D^\alpha \Gamma_\alpha + \beta \Gamma_\alpha \Gamma^\alpha = 0,\tag{9}$$

where β is an arbitrary constant. For this gauge choice the gauge-fixing and ghost terms can be written as

$$\Sigma_{gf+gh}^{NL} = \int dv \left[B(D^\alpha \Gamma_\alpha + \beta \Gamma_\alpha \Gamma^\alpha) + \widehat{C} D^2 C + 2\beta \widehat{C} \Gamma_\alpha D^\alpha C \right].\tag{10}$$

The effective action for the $N = 1$ Abelian super-Chern-Simons theory in such non-linear gauge is given by

$$\Sigma_{eff}^{NL} = \Sigma_{inv} + \Sigma_{gf+gh}^{NL},\tag{11}$$

which is also invariant under the same set of BRST transformations (8).

III. $N = 1$ ABELIAN SUPER-CHERN-SIMONS MODEL IN BV FORMULATION

In this section, we establish the theory in BV formulation. For this purpose, we need antifields corresponding to fields having opposite statistics. In terms of field and antifields, the generating functional for the $N = 1$ Abelian super-Chern-Simons theory in Landau gauge is defined by

$$Z_L = \int \mathcal{D}\Phi \, e^{i(\Sigma_{inv} + \Gamma^{\alpha*} D_\alpha C + \widehat{C}^* B)},\tag{12}$$

where $\Gamma^{\alpha*}$ and \widehat{C}^* are antifields corresponding to the Γ^α and \widehat{C} fields with opposite statistics. The above generating functional can further be written in compact form as

$$Z_L = \int \mathcal{D}\Phi \, e^{iW_{\Psi^L}[\Phi, \Phi^*]},\tag{13}$$

where $W_{\Psi^L}[\Phi, \Phi^*]$ is an extended quantum action for the $N = 1$ Abelian super-Chern-Simons theory in the Landau gauge and Φ^* refers to the antifields generically corresponding to the collective field $\Phi(\equiv \Phi_\pm, \bar{\Phi}_\pm, \Gamma_\alpha, C, \widehat{C}, B)$. The extended quantum action, $W_{\Psi}[\Phi, \Phi^*]$, satisfies a certain rich mathematical relation, the so-called quantum master equation [19], which is given by

$$\Delta e^{iW_{\Psi}[\Phi, \Phi^*]} = 0, \quad \Delta \equiv \frac{\partial_r}{\partial \Phi} \frac{\partial_r}{\partial \Phi^*} (-1)^{\epsilon+1}.\tag{14}$$

In other words, the extended quantum action W_{Ψ} is the solution of the quantum master equation. The antifields Φ^* for a general gauge theory can be evaluated from the expression of the gauge-fixed fermion. For the $N = 1$ super-Chern-Simons theory in Landau gauge the antifields are computed with the help of the following gauge-fixed fermion $\Psi^L = \widehat{C} D^\alpha \Gamma_\alpha$:

$$\Phi_{1\pm}^* = \frac{\delta \Psi^L}{\delta \Phi_\pm} = 0,$$

$$\begin{aligned}
\bar{\Phi}_{1\pm}^* &= \frac{\delta\Psi^L}{\delta\Phi_{\pm}} = 0, \\
\Gamma_{1\alpha}^* &= \frac{\delta\Psi^L}{\delta\Gamma_{\alpha}} = -D^{\alpha}\hat{C}, \\
\hat{C}_1^* &= \frac{\delta\Psi^L}{\delta\hat{C}} = D^{\alpha}\Gamma_{\alpha}, \\
C_1^* &= \frac{\delta\Psi^L}{\delta C} = 0.
\end{aligned} \tag{15}$$

With these values of antifields the extended quantum action in (12) coincides with the total effective action (7). However, the gauge-fixing fermion for the non-linear gauge choice is given by

$$\Psi^{NL} = \hat{C}(D^{\alpha}\Gamma_{\alpha} + \beta\Gamma_{\alpha}\Gamma^{\alpha}). \tag{16}$$

The antifields with the help of the above gauge-fixing fermion for the non-linear gauge are calculated as:

$$\begin{aligned}
\Phi_{2\pm}^* &= \frac{\delta\Psi^{NL}}{\delta\Phi_{\pm}} = 0, \\
\bar{\Phi}_{2\pm}^* &= \frac{\delta\Psi^{NL}}{\delta\bar{\Phi}_{\pm}} = 0, \\
\Gamma_{2\alpha}^* &= \frac{\delta\Psi^{NL}}{\delta\Gamma_{\alpha}} = -D^{\alpha}\hat{C} + 2\beta\hat{C}\Gamma_{\alpha}, \\
\hat{C}_2^* &= \frac{\delta\Psi^{NL}}{\delta\hat{C}} = D^{\alpha}\Gamma_{\alpha} + \beta\Gamma_{\alpha}\Gamma^{\alpha}, \\
C_2^* &= \frac{\delta\Psi^{NL}}{\delta C} = 0.
\end{aligned} \tag{17}$$

Similar to the linear gauge case, the generating functional for the $N = 1$ Abelian Super-Chern-Simons theory in non-linear gauge can be written in compact form as

$$Z_{NL} = \int \mathcal{D}\Phi e^{iW_{\Psi^{NL}}[\Phi, \Phi^*]}, \tag{18}$$

where $W_{\Psi^{NL}}[\Phi, \Phi^*]$ is an extended quantum action (a solution of the quantum master equation) in non-linear gauge.

IV. SOLUTIONS OF THE QUANTUM MASTER EQUATION: FIELD/ANTIFIELD DEPENDENT SYMMETRY

In this section, we analyse the field/antifield dependent BRST transformation which is characterized by the field/antifield dependent BRST parameter. To achieve the goal, we first define the usual BRST transformation for the generic field $\Phi_{\alpha}(x)$ written compactly as

$$\Phi'_{\alpha}(x) - \Phi_{\alpha}(x) = \delta_b\Phi_{\alpha}(x) = s_b\Phi_{\alpha}(x)\eta = \mathcal{R}_{\alpha}(x)\eta, \tag{19}$$

where $\mathcal{R}_{\alpha}(x)(s_b\Phi_{\alpha}(x))$ is the Slavnov variation of the field $\Phi_{\alpha}(x)$ satisfying $\delta_b\mathcal{R}_{\alpha}(x) = 0$. Here the infinitesimal transformation parameter η is a Grassmann parameter.

Now, we propose the field/ antifield dependent BRST transformation (as the discussed field-dependent BRST transformation in Ref. [39]) defined as

$$\delta_b\Phi_{\alpha}(x) = \Phi'_{\alpha}(x) - \Phi_{\alpha}(x) = \mathcal{R}_{\alpha}(x)\eta[\Phi, \Phi^*], \tag{20}$$

where the Grassmann parameter $\eta[\Phi, \Phi^*]$ is the field/antifield dependent parameter of the transformation. The field/antifield dependent BRST transformation for the $N = 1$ Abelian super-Chern-Simons theory is constructed by making the transformation parameter of (8) field/antifield dependent as

$$\begin{aligned}\delta_b \Phi_\pm &= \pm i C \Phi_\pm \eta[\Phi, \Phi^*], & \delta_b \bar{\Phi}_\pm &= \mp i C \bar{\Phi}_\pm \eta[\Phi, \Phi^*], \\ \delta_b \Gamma_\alpha &= D_\alpha C \eta[\Phi, \Phi^*], & \delta_b \widehat{C} &= B \eta[\Phi, \Phi^*], \\ \delta_b C &= 0, & \delta_b B &= 0.\end{aligned}\tag{21}$$

Though the field/antifield dependent BRST transformation is not nilpotent in nature, it is the symmetry of the action. However it does not leave the generating functional invariant. Under field/antifield dependent BRST transformation (21) the generating functional given in (13) transforms as

$$\begin{aligned}\delta_b Z_L &= \int \mathcal{D}\Phi \text{ (sDet } J[\Phi, \Phi^*]) e^{i W_{\Psi L}[\Phi, \Phi^*]}, \\ &= \int \mathcal{D}\Phi_i e^{i(W_{\Psi L}[\Phi, \Phi^*] - i \text{Tr} \ln J[\Phi, \Phi^*])}.\end{aligned}\tag{22}$$

Furthermore, we calculate the Jacobian matrix of the field/antifield dependent BRST transformation as follows

$$\begin{aligned}J_\alpha^\beta[\Phi, \Phi^*] &= \frac{\delta \Phi'_\alpha}{\delta \Phi_\beta} = \delta_\alpha^\beta + \frac{\delta \mathcal{R}_\alpha(x)}{\delta \Phi_\beta} \eta[\Phi, \Phi^*] + \mathcal{R}_\alpha(x) \frac{\delta \eta[\Phi, \Phi^*]}{\delta \Phi_\beta}, \\ &= \delta_\alpha^\beta + \mathcal{R}_\alpha^\beta(x) \eta[\Phi, \Phi^*] + \mathcal{R}_\alpha(x) \eta^\beta[\Phi, \Phi^*].\end{aligned}\tag{23}$$

Making use of the nilpotency property of the BRST transformation (i.e. $s_b^2 = 0$) and (23), we compute

$$\text{sTr} \ln J[\Phi, \Phi^*] = -\ln(1 + s_b \eta[\Phi, \Phi^*]),\tag{24}$$

where $\eta[\Phi, \Phi^*]$ exists up to linear orders only because of its anticommuting nature. Therefore, the Jacobian of the arbitrary field/antifield dependent BRST transformation is given by

$$\text{sDet} J[\Phi, \Phi^*] = \frac{1}{1 + s_b \eta[\Phi, \Phi^*]}.\tag{25}$$

Now, with this identification of the Jacobian the expression (22) simplifies as

$$\delta_b Z_L = \int \mathcal{D}\Phi e^{i(W_{\Psi L}[\Phi, \Phi^*] + i \ln(1 + s_b \eta[\Phi, \Phi^*]))},\tag{26}$$

which is nothing but the generating functional for the super-Chern-Simons theory having extended action $W_{\Psi L}[\Phi, \Phi^*] + i \ln(1 + s_b \eta[\Phi, \Phi^*])$ where the extra piece is due to the Jacobian contribution. We specifically choose the field/antifield dependent transformation parameter as follows

$$\eta[\Phi, \Phi^*] = \widehat{C} B^{-1} \left(e^{-i s_b [\widehat{C}(\widehat{C}_2^* - \widehat{C}_1^*)]} - 1 \right).\tag{27}$$

Now, we calculate the Jacobian contribution for this choice of the field-dependent transformation parameter, which leads to

$$-\ln(1 + s_b \eta[\Phi, \Phi^*]) = i[s_b \widehat{C}(\widehat{C}_2^* - \widehat{C}_1^*) + \widehat{C}(s_b \widehat{C}_2^* - s_b \widehat{C}_1^*)].\tag{28}$$

Inserting the above value in (26) we get,

$$\begin{aligned}\delta_b Z_L &= \int \mathcal{D}\Phi e^{i(W_{\Psi L}[\Phi, \Phi^*] + s_b \widehat{C}(\widehat{C}_2^* - \widehat{C}_1^*) + \widehat{C}(s_b \widehat{C}_2^* - s_b \widehat{C}_1^*))}, \\ &= \int \mathcal{D}\Phi e^{i(W_{\Psi L}[\Phi, \Phi^*] + \beta B \Gamma_\alpha \Gamma^\alpha + 2\beta \widehat{C} \Gamma_\alpha D^\alpha C)}, \\ &= \int \mathcal{D}\Phi e^{i W_{\Psi NL}[\Phi, \Phi^*]}, \\ &= Z_{NL}.\end{aligned}\tag{29}$$

Thus, we conclude that under field/antifield dependent BRST transformation with an appropriate choice of field/antifield dependent parameter the generating functionals in linear and non-linear gauges are connected. In other words, we say that under field/antifield dependent BRST transformation the different solutions of quantum master equation can be connected. We established the results at the quantum level by using the BV formulation.

V. CONCLUSIONS

In this paper we have considered the gauge invariant model of the $N = 1$ Abelian Super-Chern-Simons theory and have analysed the theory at quantum level in different gauges, namely, in Landau and non-linear (quadratic) gauges. The nilpotent BRST transformations are demonstrated for the effective actions corresponding to both gauges. Furthermore we have analysed the theory at quantum level in BV formulation which admits a mathematically rich quantum master equation. The extended quantum actions are the solutions of such quantum master equation. Furthermore, we developed the field/antifield dependent BRST transformation characterized by the field/antifield dependent parameter. For the field/antifield dependent BRST transformation we have calculated the Jacobian matrix explicitly. Remarkably, we have found that under field/antifield dependent BRST transformation with an appropriate choice of the transformation parameter the different solutions of the quantum master equation can be related. We have shown the results by connecting the extended quantum actions in Landau and non-linear gauges. Such results will help in clarifying the understanding of the dynamics of the theory in different gauges.

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